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Auger-like Relaxation of Inter-Landau-Level Magneto-Plasmon Excitations in the Quantised Hall Regime

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Auger relaxation in 2D strongly correlated electron gas can be represented as an Auger-like process for neutral magnetoplasmon excitations. The case of "dielectric" state with lack of free electrons (i.e. at integer filling ν) is considered. Really the Auger-like process is a coalescence of two magnetoplasmons which are converted into a single one of a different plasmon mode with zero 2D wave-vector. This event turns out to be energetically allowed for magnetoplasmons near their roton minima where the spectrum has the infinite density of states. As a result the additional possibility appears for indirect observation of the magnetorotons by means of anti-Stokes Raman scattering. We find the rate of this process employing the technique of Excitonic Representation for the relevant matrix element calculation.

Auger-type processes (APs) are believed to be the dominant inter-Landau-level electron scattering mechanism when emission of LO-phonons is suppressed off the magnetophonon resonance conditions. Auger scattering determines the population of Landau levels (LLs) in cyclotron resonance[1,2], anti-Stokes hot luminescence[3], and Integer Quantum Hall breakdown phenomena[4,5]. One-electron description of an AP is scattering of two electrons at the same LLs resulting in deexcitation of one of them to a lower LL and excitation of the other to a higher LL. If this lower LL is partially filled in the ground state of 2D electron gas (2DEG), then such a process reduces the total number of excited electrons, providing the 2DEG relaxation. This simple picture is based on LL equidistance and seems to correspond to real situation such as in experiments at $\nu < 1$[1]) or in the case of large LL numbers of initially excited states[4,5]. On the other hand it fails when Coulomb corrections to energy of a free electron are significant and depend on LL number. Moreover, near an integer ν the deficiency of unoccupied states in the almost filled LL leads to the conclusion that the usual AP relaxation would become very rare as it would be a result of three-particle collisions among two excited electrons and an effective hole (unoccupied state at the LL filled in ground state).

It is meanwhile well known that strong Coulomb correlations in the Quantum Hall regime renormalize drastically the 2DEG excitation spectrum. The electron promoted from n-th LL to $(n+m)$-th one and the effective hole left at the n-th LL interact with each other; hence they should be considered as a collective excitation. For integer filling the spectrum, being of dielectric type (with Zeeman gap $|g\mu_b B|$ for an odd ν, and with cyclotron gap $\hbar\omega_c$ if ν is even), is represented by chargeless excitations, namely: intra-LL spin-waves ($m=0$), inter-LL cyclotron excitations without spin-flip (so-called magnetoplasmons (MPs) with $m \neq 0$), and those with spin-flip[7,8]. In this representation an Auger-type process could be realized as a conversion of two MPs with energy in the vicinity of $m\hbar\omega_c$ into one MP in the vicinity of $2m\hbar\omega_c$.

The lowest energy MP with $m=1$ has pronounced roton type minimum in the energy dependence $\epsilon(q)$ on the 2D wave vector $\mathbf{q}$.[7,8,9,10]. Near this minimum the density of states is infinite and this is the reason why the corresponding excitations, magnetorotons, were detected by means of resonant combination backscattering[11,12,13] though this detection is only possible due to breakdown of wave-vector conservation (see discussion in Refs.13,14). In the measured signal only one other peak of the same MP mode just close to $\hbar\omega_c$ is observed. It corresponds to the MP with $\mathbf{q}$ near the origin, and satisfying the momentum conservation this peak is more intensive even though the MPs at $\mathbf{q}=0$ have much lower density of states[10]. Important for a coalescence of two MPs is the energetic possibility of their conversion into some other excitation. We see that this process being allowed for magnetorotons is forbidden for the MPs with $\mathbf{q}=0$, since the energy of the final is essentially higher than $2\hbar\omega_c$ due to Coulomb corrections[8]. Analogously the coalescence is forbidden for two MPs which are in the other "suspicious" phase region, namely, near the $\epsilon(q)$ maximum (not observed experimentally as yet) where the density of states is also infinite. The energy of one "two-cyclotron" MP is essentially lower than the combined energy of such two MPs near the maxima. Thus, the mentioned experimental detection of magnetorotons and the energetic possibility of the considered process are the reasons explaining our special interest in the MPs coalescence near their roton minima. Moreover, it is preferable to find out the generated "two-cyclotron" MP in the state with small 2D wave-vector, because in this case the generated MP could be detected by anti-Stokes Raman

scattering like in the experiments of Refs.[11,12,13]. This is why we will present more detailed results exactly for this case. We calculate the decay rate of such Auger-like process.

We solve this problem for the case of "strong magnetic field", i.e. in the lowest-order approximation in the small parameter $E_c/\hbar\omega_c$, where $E_c = e^2/\kappa l_B$ is a characteristic Coulomb energy for electron-electron (e-e) interaction in 2DEG, l_B being the magnetic length, and κ the effective dielectric constant. (For $B = 10\,\mathrm{T}$ $\hbar\omega_c = 17.3\,\mathrm{meV}$, $l_B = 8.1\,\mathrm{nm}$ and $E_c = 14\,\mathrm{meV}$). It is well known that in this approximation the problem of two-particle excitation spectrum for $\nu = integer$ can be solved exactly[7,8]. Now, our task is the calculation of the transition matrix element.

Further we employ the so-called excitonic representation (ER), which is very advantageous for excitations from a filled LL. Let us label a certain one-electron state characterized by its LL and spin sublevel by $a = (n_a, \sigma_a)$. Then the excitations may be considered as effective excitons with energies

$$\epsilon_{ab}(q) = \hbar\omega_c m + |g\mu_b B|\delta S_z + \mathcal{E}_{ab}(q), \qquad (1)$$

where $m = n_b - n_a$, $\quad \delta S_z = \sigma_b - \sigma_a$, $\quad (\sigma_a,\ \sigma_b = \pm 1/2)$, and the energy $\mathcal{E}_{ab}$ has a Coulomb origin. It is of the order of or smaller than E_c.

We restrict ourselves only to the case of $\nu = 1$ considering only MPs with $n_a = 0$ and $n_b = 1, 2$ and with $\sigma_a = \sigma_b = +1/2$. In this case we change the subscript ab in Eq.(1) to 01 or 02, respectively. The analytical and numerical calculation of the excitation spectra of these 01 and 02 MPs are presented in[8] in the strict 2D limit (S2DL) when the thickness of the 2DEG d satisfies the condition $d \ll l_B$. In fact the spectra depend on d but their shape do not change qualitatively. The function $\mathcal{E}_{01}(q)$ has a roton minimum at $q = q_0 \approx 1.92/l_B$:

$$\mathcal{E}_{01}(q) = \varepsilon_0 + (q - q_0)^2/2M, \quad |q - q_0| \ll q_0\,, \qquad (2)$$

where in S2DL $M^{-1} \approx 0.28\,E_c l_B^2$ and $\varepsilon_0 \approx 0.15\,E_c$. The dependence $\mathcal{E}_{02}(q)$ is also nonmonotonic, but in the range $0 < ql_B < 2.5$ does not change more than $0.07\,E_c$. Of special importance is the difference $\delta = \mathcal{E}_{02}(0) - 2\varepsilon_0$, which "casually" is numerically small in the scale of E_c, namely in S2DL $\delta \approx 0.019\,E_c \simeq 3 \div 4$ K for $B = 10 \div 20\,\mathrm{T}$, but is positive[19]. The desired matrix element of the considered conversion is

$$\mathcal{M}(\mathbf{q}_1, \mathbf{q}_2) = {}_{02}\langle \mathbf{q}_1 + \mathbf{q}_2; 1|H|\mathbf{q}_1, \mathbf{q}_2; 2\rangle_{01}. \qquad (3)$$

Here H is Hamiltonian, the initial state is a two 01MP state, and the final one is a one 02MP state.

The total 2DEG Hamiltonian is $H = H_0 + H_{int}$, where the Hamiltonian of the noninteracting electrons is

$$H_0 = \sum_{n,p,\sigma} \left[(n + 1/2)\hbar\omega_c - |g\mu_b B|\sigma\right] e^+_{n,p,\sigma} e_{n,p,\sigma}\,. \qquad (4)$$

Here $e_{n,p,\sigma}$ is the electron annihilation operator at n-th LL having σ as the $\hat{z}$-component of spin, $p = k_y$ is the intra-level Landau gauge quantum number. Within the framework of strong magnetic field approximation it is enough to keep in the interaction Hamiltonian, H_{int}, only the terms which conserve cyclotron part of the energy, or in other words, the terms which commute with H_0. The Coulomb part of the Hamiltonian may therefore be written in the form

$$H_{int} = \mathcal{N}^{-1} \sum_{\substack{p,p',\mathbf{q}\\ n,m,l,k,\sigma_1,\sigma_2}} V_{nmlk}(q) \exp\left[iq_x(p' - p)\right] \cdot e^+_{n,p+q_y,\sigma_1} e^+_{m,p',\sigma_2} e_{l,p'+q_y,\sigma_2} e_{k,p,\sigma_1}\,, \qquad (5)$$

which provides automatically the cyclotron energy conservation rule $n + m = l + k$, because

$$V_{nmlk}(q) = (2\pi)^{-1} V(q) h_{nk}(\mathbf{q}) h^*_{lm}(\mathbf{q}) \delta_{n+m,l+k}. \qquad (6)$$

We use now dimensionless length and wave-vectors measured in the units of l_B and l_B^{-1}. $\mathcal{N} = L^2/2\pi l_B^2$ is the total number of magnetic flux quanta in the normalization area L^2, and $V(q)$ is the 2D Fourier component of the Coulomb potential averaged with the wave function in the $\hat{z}$ direction (so that in S2DL: $V(q) = 2\pi E_c/q$), and

$$h_{nk}(\mathbf{q}) = \int_{-\infty}^{+\infty} dx \chi_n(x + q_y/2) e^{iq_x x} \chi_k(x - q_y/2) = \left[\frac{\min(n,k)!}{\max(n,k)!}\right]^{1/2} \left[\frac{iq_x + q_y\,\mathrm{sign}\,(n-k)}{\sqrt{2}}\right]^{|n-k|} \cdot e^{-q^2/4} L^{|n-k|}_{\min(n,k)}(q^2/2)$$

$\chi_n(x)$ is the normalized n-th harmonic oscillator function, L_n^j is Laguerre polynomial.

Now we define in ER the states in the matrix element (3) in order to calculate the last one. Let a be the filled LL, i.e. in our particular case $a = (0, 1/2)$. We designate $a_p \equiv e_{n_a,p,\sigma_a}$ while $b_p \equiv e_{n_b,p,\sigma_b}$ for every other one-electron state b. The ER means a replacement of operators $e^+_{n,p,\sigma}$ and $e_{n,p,\sigma}$ by a set of inter-LL "excitonic" creation and annihilation operators for $a \neq b$ (i.e. $n_b \neq n_a$, or $\sigma_a \neq \sigma_b$)

$$\mathcal{Q}^+_{ab\mathbf{q}} = \frac{1}{\sqrt{\mathcal{N}}} \sum_p e^{-iq_x p} b^+_{p+\frac{q_y}{2}}\, a_{p-\frac{q_y}{2}}\,, \;\; \mathcal{Q}_{ab\mathbf{q}} = \mathcal{Q}^+_{ba\,-\mathbf{q}}, \qquad (7)$$

and intra-LL "displacement" operators $A_{\mathbf{q}}$ and $B_{\mathbf{q}}$ (see Ref.[15]). We do not write here the latter ones, because they, being required for total ER of Hamiltonian (5), are not used directly for matrix element (3) calculation.

Some commutation rules for operators (7) are the same as the ones obtained in Ref.[15] minding the case $a = (n, 1/2)$, $b = (n, -1/2)$. We derive the additional ones considering $a \neq b \neq c$:

$$[\mathcal{Q}^+_{bc\mathbf{q_1}}, \mathcal{Q}_{ab\mathbf{q_2}}] = 0, \;\; [\mathcal{Q}^+_{bc\mathbf{q_1}}, \mathcal{Q}^+_{ab\mathbf{q_2}}] = \frac{e^{-i\Theta_{12}}}{\mathcal{N}^{1/2}} \mathcal{Q}^+_{ac\,\mathbf{q_1}+\mathbf{q_2}}. \qquad (8)$$

Here $\Theta_{12} = \Theta(\mathbf{q}_1, \mathbf{q}_2) = (\mathbf{q}_1 \times \mathbf{q}_2)_z/2 = \frac{1}{2} q_1 q_2 \sin\alpha$, where α is an angle between $\mathbf{q}_2$ and $\mathbf{q}_1$. Note that the

considered operators were employed earlier in some other form as applied to "valley-wave" excitations[16] and, also, to spin-waves[15,17], when $m = 0$, $|\delta S_z| = 1$.

The operator $\mathcal{Q}^+_{ab\mathbf{q}}$ creates a abMP: $|\mathbf{q};1\rangle_{ab} = \mathcal{Q}^+_{ab\mathbf{q}}|0\rangle$. Here $|0\rangle$ is the ground state where the level a is fully occupied, whereas b is empty: $a_p^+|0\rangle = b_p|0\rangle \equiv 0$. This is equivalent to identities $A^+_{\mathbf{q}}|0\rangle \equiv \delta_{0,\mathbf{q}}|0\rangle$ and $B^+_{\mathbf{q}}|0\rangle \equiv \mathcal{Q}_{ab\mathbf{q}}|0\rangle \equiv 0$.

The choice of the state b depends on a type of problem. In our case the states entering the matrix element (3) are

$$|\mathbf{q};1\rangle_{02} = \mathcal{Q}^+_{02\,\mathbf{q}}|0\rangle, \quad |\mathbf{q}_1,\mathbf{q}_2;2\rangle_{01} = \mathcal{Q}^+_{01\mathbf{q}_1}\mathcal{Q}^+_{01\mathbf{q}_2}|0\rangle, \quad (9)$$

As above 01 and 02 stand for ab with $a = (0,1/2)$ and $b = (1,1/2)$ or $b = (2,1/2)$ respectively. These states are orthogonal and are eigenstates of the Hamiltonian H in the limit $\mathcal{N} \to \infty$, i.e.

$$\begin{aligned} &H_{int}|\mathbf{q};1\rangle_{02} = [E_0 + \mathcal{E}_{02}(q)]|\mathbf{q};1\rangle_{02} + \cdots, \\ &H_{int}|\mathbf{q}_1,\mathbf{q}_2;2\rangle_{01} = \\ &\qquad [E_0 + \mathcal{E}_{01}(q_1) + \mathcal{E}_{01}(q_2)]|\mathbf{q}_1,\mathbf{q}_2;2\rangle_{01} + \cdots \;, \end{aligned}$$

where E_0 is the Coulomb ground state energy ($H_{int}|0\rangle = E_0|0\rangle$) and the dots correspond to some states having a norm of the order of $E_c/\mathcal{N}$. The states (9) are the correct initial and final states in the scattering problem for a low density gas of MPs, but the scattering matrix element (3) has to be calculated with higher accuracy, since $\mathcal{M} \sim \mathcal{N}^{-1/2}$.

Instead of the value (3) it is more convenient to calculate the conjugate one $\mathcal{M}^*$ substituting in Eq. (3) the expressions (9). After one has done the ER transformation of the Hamiltonian (5) in terms of operators (7) together with $A_{\mathbf{q}}$ and $B_{\mathbf{q}}$. then taking into account the properties of the ground state $|0\rangle$ and the commutation rules (8) one finds that the only term of Hamiltonian which contributes to the matrix element (3) is

$$\sum_{\mathbf{q}} V_{1120}(q)\mathcal{Q}^+_{01\mathbf{q}}\mathcal{Q}_{12\mathbf{q}}\,.$$

Using again as tools the properties of operators (7) and of the state $|0\rangle$ we obtain for $\mathbf{q}_1 \neq \mathbf{q}_2$

$$\begin{aligned} \mathcal{M}(\mathbf{q}_1,\mathbf{q}_2) = \mathcal{N}^{-1/2}\big[&u(q_1)e^{-i\Theta_{12}} + u(q_2)e^{i\Theta_{12}} \\ &- v(q_1)e^{-i\Theta_{12}} - v(q_2)e^{i\Theta_{12}}\big], \quad (10) \end{aligned}$$

where

$$\begin{aligned} u(q) &= (2^{5/2}\pi)^{-1}q^2V(q)[2-(ql_B)^2/2]e^{-(ql_B)^2/2}, \\ v(q) &= l_B^2\int_0^\infty dppu(p)J_0(pql_B^2)\,. \end{aligned}$$

We returned here to dimensional quantities (also suitable redefinition is $V(q) \to l_B^2V(q)$). In S2DL one can find analytic expression of $v(q)$, involving polynomials, exponentials, and modified Bessel functions.

The depopulation rate of a 01MPs due to their coalescence is

$$\begin{aligned} \mathcal{R} = \frac{1}{2}\sum_{\mathbf{q}_1,\mathbf{q}_2}\frac{2\pi}{\hbar}&\left|\mathcal{M}(\mathbf{q}_1,\mathbf{q}_2)\right|^2\overline{n}(\mathbf{q}_1)\overline{n}(\mathbf{q}_2)\,\cdot \\ &\delta\left[\mathcal{E}_{01}(\mathbf{q}_1) + \mathcal{E}_{01}(\mathbf{q}_2) - \mathcal{E}_{02}(\mathbf{q}_1+\mathbf{q}_2)\right] \qquad (11) \end{aligned}$$

where $\overline{n}(\mathbf{q})$ are occupation numbers of 01MPs. We consider the occupancy for 02MPs to be small and do not take into account corresponding stimulated processes and 02MP decay. (Note that the terms with $\mathbf{q}_1 = \mathbf{q}_2$ give no essential contribution to this sum).

Because of the mentioned reasons, we will consider a special situation when the 01MPs occupy states near the magnetoroton minima and calculate the rate of depopulation due to creation of 02MPs only with small q. These 02MPs can be detected by anti-Stokes Raman scattering like in experiments[11,12,13]. For this purpose we have to sum in Eq.(11) with the restriction $|\mathbf{q}_1+\mathbf{q}_2| < \tilde{q}$ and we will show later that $\tilde{q} \ll l_B^{-1}$. Under these assumptions we can put in $|\mathcal{M}|^2$ $\mathbf{q}_1 = -\mathbf{q}_2$ and $|\mathbf{q}_1| = |\mathbf{q}_2| = q_0$. We also assume that $\overline{n}(\mathbf{q}) = \overline{n}$ can be considered to be constant as long as due to energy conservation we are dealing with the a narrow band determined by inequalities: $\varepsilon_0 < \mathcal{E}_{01}(q) < \mathcal{E}_{02}(\tilde{q}) - \varepsilon_0$. Using this simplifications and replacing summation by integration $\sum_{\mathbf{q}} = \mathcal{N}l_B^2\int d^2q/(2\pi)$ we find the rate of 02MP creation with $|\mathbf{q}| < \tilde{q}$ per unit area to be

$$\frac{\mathcal{R}(|\mathbf{q}|<\tilde{q})}{L^2} = \frac{\overline{n}^2l_B^2q_0}{2\hbar}[u(q_0)-v(q_0)]^2\left(\frac{M}{\delta}\right)^{1/2}\tilde{q}^2. \quad (12)$$

The question to be considered is the role of the random impurity potential $U(\mathbf{r})$ which was neglected in the above calculations. The distance between an excited electron and a hole in real space is $l_B^2\mathbf{q}\times\hat{z}$ (see Refs.[7,8,9]). Assuming $U(\mathbf{r})$ to be smooth (correlation length $\Lambda \gg l_B$) one can find that the energy correction for a MP with the wave vector $\mathbf{q}$. In the dipole approximation it is $\delta\mathcal{E}(\mathbf{q},\mathbf{r}) = -\hbar\mathbf{q}\mathbf{v}_d$ for any abMP, where $\mathbf{v}_d = (\hat{z}\times\nabla U(\mathbf{r}))l_B^2/\hbar$ is the drift velocity. This additional energy leads to inhomogeneous broadening of the MP energy. One can see that the random potential correction plays no significant role if

$$|d\mathcal{E}_{ab}/dq| \gg l_B^2\,|\nabla U| \qquad (13)$$

which means that the electron-hole Coulomb interaction is stronger than the force the electron and the effective hole are subjected to in the random electric potential. Evidently the other meaning of this condition is that the exciton velocity has to be greater than the drift velocity in the external field[19]. Alternatively, we have two independent quasiparticles, electron and hole, whose motion is determined mainly by the random potential and the e-e interaction has to be considered only as a perturbation[6]. For $q \simeq l_B^{-1}$ one can estimate the inhomogeneous broadening $\delta\mathcal{E} \simeq \Delta(l_B/\Lambda)$, where Δ is the random potential amplitude (i.e. $\nabla U \sim \Delta/\Lambda$). With typical values $\Lambda = 50$nm and $\Delta = 1$meV one finds $\delta\mathcal{E} = 0.2$meV. This is small compared to the width of the MP band and small compared to ϵ_0 but of the same order as the energy δ relevant for energy conservation. At the same time, since in the dipole approximation the inhomogeneous broadening of the level $\mathcal{E}_{ab}(\mathbf{q})$ do not depend on ab it gives no contribution to the delta-function argument in Eq.(11). Higher order corrections to $\delta\mathcal{E}$ are of the order of $\Delta(l_B/\Lambda)^2 \simeq 0.04$ meV and small compared even

to δ. As a result we conclude that the role of the random potential is negligible compared to the e-e interactions.

However the role of the random potential is crucial in determining the cutoff $\tilde{q}$. The momentum of a 02MP detected by anti-Stokes Raman scattering is defined from momentum conservation as $\mathbf{q}=\mathbf{k}_{2\|}-\mathbf{k}_{1\|}$, where $\mathbf{k}_{1\|}$ and $\mathbf{k}_{2\|}$ are the "in-plane" wave vector components of the incident and scattered photons. In the case of no disorder the cutoff $\tilde{q}$ is defined by the uncertainty of $\mathbf{k}_{2\|}-\mathbf{k}_{1\|}$, i.e. by the spectral resolution and the geometry of the optical experiment. This uncertainty is $<10^4\,\mathrm{cm}^{-1}$ according to Refs.[11,12,13] and the cutoff $\tilde{q}$ actually comes from the disorder which violates momentum conservation.

In the approximation of S2DL one may estimate $d\mathcal{E}_{02}/dq \simeq E_c q^2 l_B^3$ for $ql_B \ll 1$[18], and the uncertainty of q due to disorder can be found from Eq.(13) giving $\tilde{q} \simeq (\Delta/E_c)^{1/2}(\Lambda l_B)^{-1/2}$. This value does not depend on the magnetic field and for the used numerical parameters $\tilde{q} \sim 10^5\,\mathrm{cm}^{-1}$. The substitution into Eq.(12) gives

$$\mathcal{R}(|\mathbf{q}|<\tilde{q})/L^2 \sim 0.05 \cdot \frac{\overline{n}^2\Delta}{\hbar l_B \Lambda} \quad \propto \quad B^{1/2} \qquad (14)$$

(it is taken into account that $u(q_0)-v(q_0) \approx -0.062E_C$).

Now let us estimate the total decay of 01MPs supposing that the most of them are concentrated in the vicinity of the roton minima. Generally, a more complicated summation (12) has to be fulfilled in this case, because the allowed phase region where 02MPs can be generated is not small. Indeed, the very weak dependence $\mathcal{E}_{02}(q)$ in its initial spectrum portion leads to the only condition $q \lesssim l_B^{-1}$ for allowed O2MP wave-vectors. However to obtain the approximate total rate of the coalescing 01MPs the formula (14) can be exploited again. Estimating the 01MP density near their roton minima as $N \simeq \overline{n} q_0 (2M\delta\mathcal{E})^{1/2}$ (because the roton minima broadening due to inhomogeneity is $|\mathbf{q}-\mathbf{q}_0| \sim (2M\delta\mathcal{E})^{1/2}$) and setting dN/dt equal to decay rate (14) with $\tilde{q} \sim l_B^{-1}$ we find the characteristic relaxation time $\tau=\overline{n}dt/d\overline{n}$ which turns out to be

$$\tau \sim 10^2\hbar(\Delta l_B/E_C^3\Lambda)^{1/2}/\overline{n} \sim 1/\overline{n}\,\mathrm{ps}$$

(therefore $\tau \propto 1/B$). This value should be for real experiments compared with time characteristics of other possible relaxation channels, for example when the conditions of magnetophonon resonance are satisfied.

The value $\overline{n}$ remains indefinite because it depends on the specific manner of 01MPs excitation. We think the photoluminescence excitation technique is likely to be more appropriate for it, as far as therewith the excitation would occur in two independent steps: namely, by generation of an electron at the 1-th LL and a hole in the valence band, and by recombination of some electron from the filled LL with the hole. As a result 01MPs with various $\mathbf{q}$-s can appear. This technique should be more effective for magnetoroton excitation in comparison with Refs.[11,12,13] though in itself it does not permit to detect the magntorotons. Nevertheless, if one simultaneously could find 02MPs by means of anti-Stokes Raman scattering or by means of hot luminescence from the 2-nd LL, it would be an indirect confirmation of the presence of 01MPs near their roton minima. Note also that the appropriate consideration of kinetic relations shows that the occupation number for 02MPs could be expected to be of the order of $\overline{n}^2$ once the quasi-equilibrium $2\times$01MP$\leftrightarrow$02MP is established.

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[18] More accurately the analytical expression for $\mathcal{E}_{02}(q)/E_c$ (see Ref. [8]) contains the lowest order term $c(ql_q)^2$ near $q=0$, but the factor c is in S2DL numerically small.
[19] If δ turns out to be negative, then the direct transition (i.e. without acoustic phonon participation) would be possible only for generation 02MP with rather large wave-vector, which is outside of region required for detection (see text).